High School Mathematics NECAP GSE Support Materials
March 30, 2006

This document contains 71 items keyed to the NH, RI, and VT Draft High School Mathematics NECAP GSEs that were a part of a two-day item-fitting process. The purpose of item fitting was to use items to help clarify the Draft GSEs. While the items exemplify aspects of the Mathematics NECAP GSEs, they do **NOT** illustrate ALL aspects of the GSEs. Furthermore, while the majority of the items are released items from various state and national assessments, some of the items may not be suitable for on-demand statewide assessments, but are items that help exemplify the spirit of the GSEs. **Items were drawn from the following sources:** NAEP, PISA, NHEIAP, MCAS, WYCAS, VSMIT, ARTIST, PSSM, Balanced Assessment (from the MARS web site), and New Standards. Appendix A contains web links to the primary pages where the samples items are located. Many of these sites contain additional materials such as scoring rubrics, exemplars, answers, and performance data. Appendix B contains the Depth of Knowledge descriptors based on Norman Webb's work and Appendix C contains preliminary Depth of Knowledge levels for each GSE. Appendix D contains the preliminary Distribution of Emphasis for the high school assessment. Appendix E contains answers and a Depth of Knowledge classification for each item. The original item numbers (from their sources) were preserved when possible for ease of locating the items from their original sources. In addition, each item is numbered sequentially in the upper right corner for ease of reference to the answers in Appendix E.

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M(N&O)–10–7		
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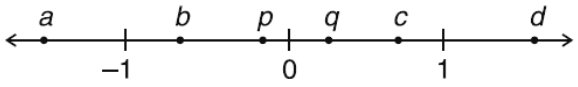
Black – No Assessment GSE at this level

M(N&O)–10–2 **Demonstrates understanding of the relative magnitude of real numbers** by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., $\sqrt{2}$, π), rational bases with integer exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.

1

Use the number line below to answer question 1.



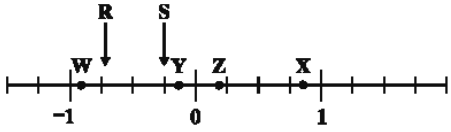
1. Which point on the number line is closest in value to $\frac{p}{q}$?

4% A. point *a*
* 43% B. point *b*
50% C. point *c*
2% D. point *d*

WYCAS Released Item – 2003

2

Use the number line below to answer question 30.

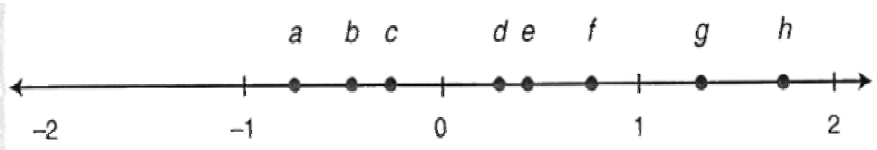


30. Which point **best** represents the product of R and S?

A. point W
B. point X
C. point Y
✱ D. point Z

NHEIAP Released Item – 2001

3



Given the points with coordinates *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *h* as shown, Which point is closest to *ab*? To|*c*|? To 1/*f*? To $\sqrt{e}$? To $\sqrt{h}$? Explain your reasoning.

National Council of the Teachers of Mathematics, Inc., *Principles and Standards for School Mathematics*, National Council of the Teacher of Mathematics, 2000.

M(N&O)–10–4 **Accurately solves problems involving** rational numbers within mathematics, across content strands, disciplines or contexts (with emphasis on, but not limited to, proportions, percents, ratios, and rates). 4

7. The population of the United States is approximately 250 million, and the national debt is approximately 4 trillion dollars. If this debt were divided equally among the population, what would be the debt, in dollars, per person?

A) 16
B) 1,600
C) 16,000
D) 1,600,000
E) 16,000,000

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8. Luis mixed 6 ounces of cherry syrup with 53 ounces of water to make a cherry-flavored drink, Martin mixed 5 ounces of the same cherry syrup with 42 ounces of water. Who made the drink with the stronger cherry flavor? 5

NAEP Released Item – 1996

Give mathematical evidence to justify your answer.

6

8 A certain car averages 28 miles per gallon. Gasoline costs $1.11 per gallon. Which of the following is closest to the number of miles the car would be expected to go on $250 worth of gasoline?

MCAS Released Item – 2004

A. 400 miles

B. 6,000 miles

C. 12,000 miles

D. 30,000 miles

7

34 The force needed to stretch a spring is directly proportional to the amount the spring is to be stretched. If a force of 100 pounds stretches a certain spring 8 inches, how much force is needed to stretch the spring 12 inches?

MCAS Released Item – 2004

A. 25 pounds

B. 50 pounds

C. 100 pounds

D. 150 pounds

M(N&O)–10–4 continued on next page

M(N&O)–10–4 **Accurately solves problems involving** rational numbers within mathematics, across content strands, disciplines or contexts (with emphasis on, but not limited to, proportions, percents, ratios, and rates).

Continued…

8

Question 14: Seal's Sleep

<< Previous

SEAL'S SLEEP

A seal has to breathe even if it is asleep in the water. Martin observed a seal for one hour. At the start of his observation, the seal was at the surface and took a breath. It then dove to the bottom of the sea and started to sleep. From the bottom it slowly floated to the surface in 8 minutes and took a breath again. In three minutes it was back at the bottom of the sea again. Martin noticed that this whole process was a very regular one.

After one hour the seal was

- ○ At the Bottom
- ○ On its way up
- ○ Breathing
- ○ On its way down

PISA Sample Item – 2003

9

13. A rectangular pool 24 feet long, 8 feet wide, and 4 feet deep is filled with water. Water is leaking from the pool at the rate of 0.4 cubic foot per minute. At this rate, how many hours will it take for the water level to drop 1 foot?

A) 4
B) 8
C) 12
D) 16
E) 32

NAEP Released Item – 1996

M(N&O)–10–4 continued on next page

M(N&O)–10–4 **Accurately solves problems involving** rational numbers within mathematics, across content strands, disciplines or contexts (with emphasis on, but not limited to, proportions, percents, ratios, and rates). 10

Continued…

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

9. One plan for a state income tax requires those persons with income of $10,000 or less to pay no tax and those persons with income greater than $10,000 to pay a tax of 6 percent only on the part of their income that exceeds $10,000.

   A person's effective tax rate is defined as the percent of total income that is paid in tax.

   Based on this definition, could any person's effective tax rate be 5 percent? Could it be 6 percent? Explain your answer. Include examples if necessary to justify your conclusions.

NAEP Released Item – 1992

11

3. A certain machine produces 300 nails per minute. At this rate, how long will it take the machine to produce enough nails to fill 5 boxes of nails if each box will contain 250 nails?

   A) 4 min
   B) 4 min 6 sec
   C) 4 min 10 sec
   D) 4 min 50 sec
   E) 5 min

NAEP Released Item – 1996

M(G&M)–10–2 **Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems** involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., **Pythagorean Theorem, Triangle Inequality Theorem**).

12

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

10. Describe a procedure for locating the point that is the center of a circular paper disk. Use geometric definitions, properties, or principles to explain why your procedure is correct. Use the disk provided to help you formulate your procedure. You may write on it or fold it in any way that you find helpful, but it will not be collected.

Note: A circular piece of paper was provided as a manipulative for this item.

NAEP Released Item – 1996

13

2. In the space below, use your ruler to draw a parallelogram that has perpendicular diagonals. Show the diagonals in your sketch.

NAEP Released Item – 1996

14

5. Which of the following is NOT a property of every rectangle?

A) The opposite sides are equal in length.
B) The opposite sides are parallel.
C) All angles are equal in measure.
D) All sides are equal in length.
E) The diagonals are equal in length.

NAEP Released Item – 1992

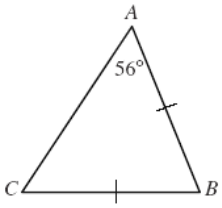
M(G&M)–10–2 continued on next page

M(G&M)–10–2 **Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems** involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., **Pythagorean Theorem, Triangle Inequality Theorem**).

Continued…

15

36 What is the measure of $\angle B$ in the figure below?

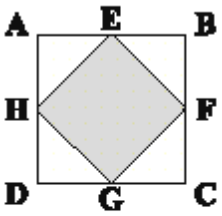


MCAS Released Item – 1994

A. 34°
B. 56°
C. 62°
D. 68°

16

Use the figure below to answer question 27.



E, F, G, and H are the midpoints of the sides of square ABCD.

NHEIAP Released Item – 2002

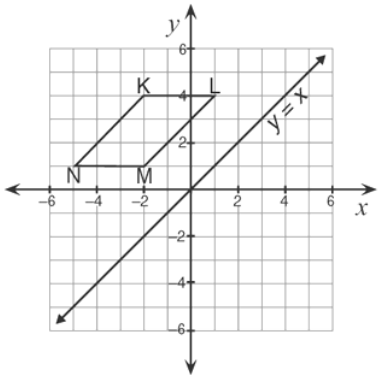
27. What is the ratio of the area of the shaded region to the total area of the unshaded regions?

A. 2 to 1
✱ B. 1 to 1
C. 1 to 2
D. 1 to 4

M(G&M)–10–4 **Applies the concepts of congruency** by solving problems on or off a coordinate plane involving reflections, translations, or rotations; or solves problems using congruency involving problems within mathematics or across disciplines or contexts.

17

Use the graph below to answer question 19.



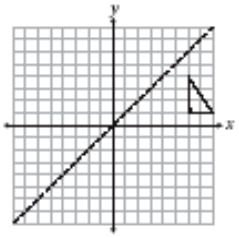
19. If figure KLMN is reflected about the line $y = x$, what will be the coordinates of K', the image of point K?

A. (4,2)
B. (4,–2)
C. (2,–4)
D. (–2,4)

NHEIAP Released Item – 2003

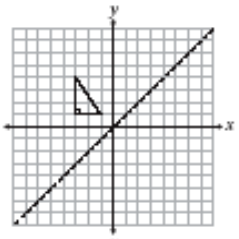
18

Use the figure below to answer question 19.

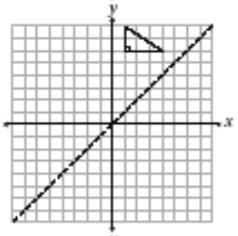


19. Which figure below shows a reflection of the above triangle through the dashed line?

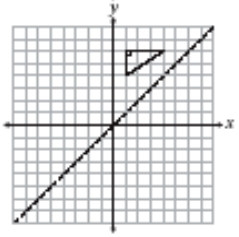
A.



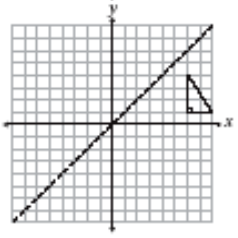
B.



C.



D.

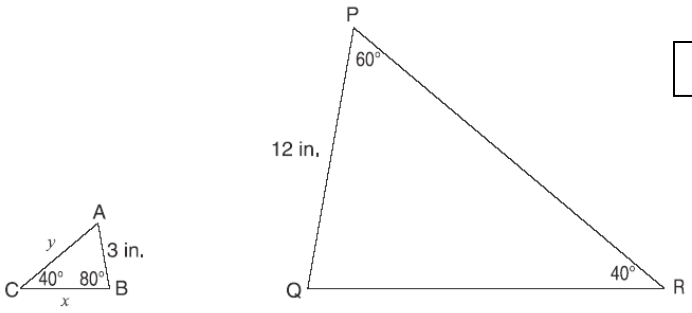


NHEIAP Released Item – 2003

M(G&M)–10–5 **Applies concepts of similarity by solving problems** within mathematics or across disciplines or contexts.

19

Use the diagram below to answer question 18.

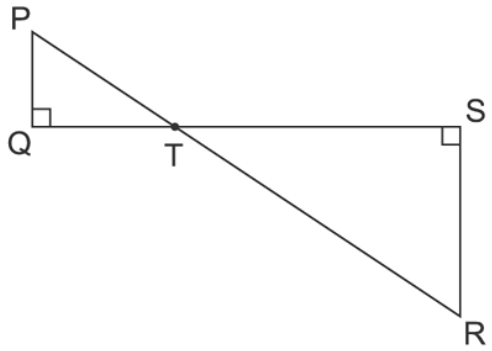


NHEIAP Released Item – 2001

18. a. Explain why triangles ABC and PQR are similar.
    b. Find the **perimeter** of each triangle in terms of *x* and *y*.
    c. What is the ratio of the **areas** of the triangles? Explain your reasoning.

20

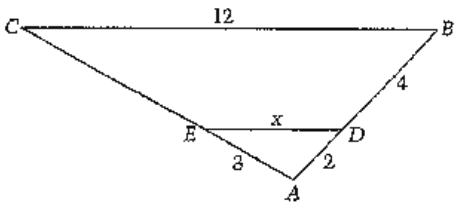
Use the figure below to answer question 21.



NHEIAP Released Item – 2003

21. In the figure, T is $\frac{2}{3}$ of the way from S to Q, and the area of ΔPQT is 6 cm^2. What is the area of ΔRST?
    A. 4 cm^2
    B. 6 cm^2
    C. 12 cm^2
    D. 24 cm^2

21



NAEP Released Item – 1996

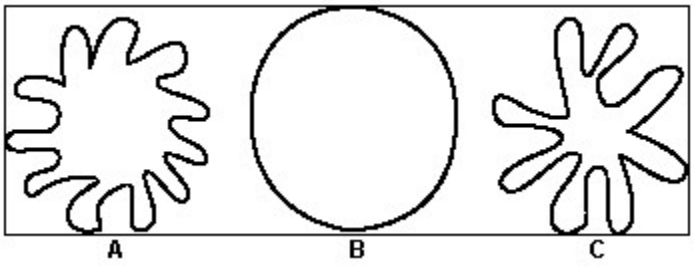
1. If triangles *ADE* and *ABC* shown in the figure above are similar, what is the value of x?

A) 4
B) 5
C) 6
D) 8
E) 10

M(G&M)–10–6 **Solves problems involving perimeter, circumference, or area** of two-dimensional figures (including composite figures) or **surface area or volume** of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts.

22

SHAPES TEXT



PISA Sample Item – 2003

Which of the figures has the largest area? Show your reasoning.

Describe a method for estimating the area of figure C.

Describe a method for estimating the perimeter of figure C.

23

41 A glass containing water is in the shape of a right circular cylinder with a radius of 3 centimeters. The height of the water in the glass is 10 centimeters.

a. What is the volume of the water in the glass? Be sure to include units of measure in your answer. Show or explain how you obtained your answer.

b. Five spherical marbles of equal size are dropped into the glass. The water in the glass rises to a height of 11 centimeters. What is the **increase** in the volume of the contents of the glass? Be sure to include units of measure in your answer. Show or explain how you obtained your answer.

c. What is the volume of one of the marbles? Be sure to include units of measure in your answer. Show or explain how you obtained your answer.

d. What is the radius of one of the marbles? Be sure to include units of measure in your answer. Show or explain how you obtained your answer.

MCAS Released Item – 2004

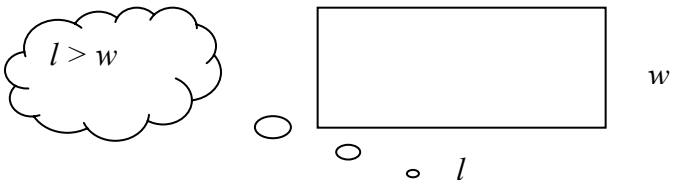
M(G&M)–10–6 continued on next page

M(G&M)–10–6 **Solves problems involving perimeter, circumference, or area** of two-dimensional figures (including composite figures) or **surface area or volume** of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts.

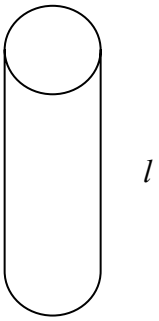
Continued…

24

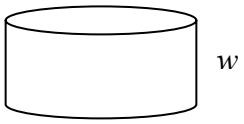
This sheet of metal was used to create the lateral surface of cylinders A and B. Would the volume of cylinder A be equal to, greater than, or less than the volume of cylinder B? Explain your answer.



RI/NH created item – 2005



Cylinder A



Cylinder B

25

37 A right circular cylindrical can is 6 inches high, and the area of its top is 36π square inches. What is the minimum number of square inches of construction paper it would take to cover the lateral surface of this can?

A. 72 sq. in.

B. 72π sq. in.

C. 432 sq. in.

D. 432π sq. in.

MCAS Released Item – 2004

M(G&M)–10–6 continued on next page

M(G&M)–10–6 **Solves problems involving perimeter, circumference, or area** of two-dimensional figures (including composite figures) or **surface area or volume** of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts.

Continued…

26

16 The length of the hypotenuse of a right triangle is 13 centimeters, and the length of one leg is 12 centimeters. What is the area of the triangle?

MCAS Released Item – 2004

27

5 The wheels on Bill's bicycle each have a radius of 35 centimeters. Which of the following is closest to the distance the bicycle moves along the ground in one complete revolution of the wheels?

A. 35 cm

B. 55 cm

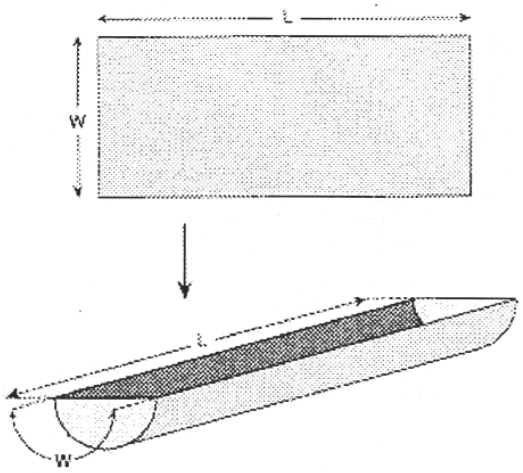
C. 110 cm

D. 220 cm

MCAS Released Item – 2004

28

A rectangular sheet of metal with length L and width W is rolled up into a gutter with semi-circular cross section. End pieces are added so that it will hold water.



Show step by step how to develop a formula for the volume of water in the gutter when it is full.

Use the symbols L and W in your formula.

Shannon, Ann. *Keeping Score.* National Research Council. 1999.

Item from New Standards™

M(G&M)–10–6 continued on next page

M(G&M)–10–6 **Solves problems involving perimeter, circumference, or area** of two-dimensional figures (including composite figures) or **surface area or volume** of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts.

Continued…

29

Jay is packing a bag for a trip. The shape of his bag is a rectangular prism. After packing he realizes that his bag is only one-half full. He reasons that he could use another bag whose dimensions are exactly one-half of the dimensions of the bag he packed. Explain how you know whether or not Jay is correct.

Item Created by RI Teachers at Prime Time Institute, 2005 (wording slightly adapted)

M(G&M)–10–7 **Makes conversions within or across systems and makes decisions concerning an appropriate degree of accuracy in problem situations** involving measurement in other GSEs.

30

7. If a 12,000-acre ranch were perfectly square, how many miles would it be on a side?
(NOTE: 640 acres = 1 sq. mile)

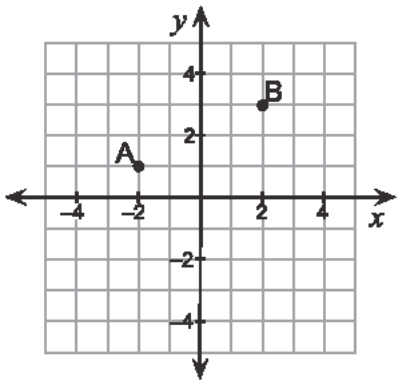
A. 4.33 miles
B. 4.69 miles
C. 6.32 miles
D. 9.38 miles

WYCAS Released Item – 2003

M(G&M)–10–9 **Solves problems on and off the coordinate plane** involving distance, midpoint, perpendicular and parallel lines, or slope.

31

Use the graph below to answer question 30.



NHEIAP Released Item – 2003

30. Which location of point C will make triangle ABC isosceles?

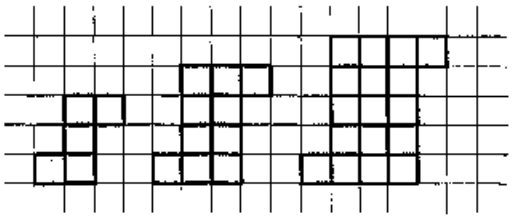
A. (–2,3)
B. (–2,2)
C. (2,–2)
D. (1,–1)

M(F&A)–10–1 **Identifies, extends, and generalizes a variety of patterns** (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

32

9. The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.



NAEP Released Item – 1996

Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

33

What is the 9th term in the sequence shown below?

2, 5, 10, 17, 26, . . .

Adapted MCAS Released Item – 2004

A. 97
B. 82
C. 71
D. 65

M(F&A)–10–1 continued on next page

M(F&A)–10–1 **Identifies, extends, and generalizes a variety of patterns** (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations.

Continued…

From the MARS web site – Balanced Assessment for the Mathematics Curriculum - High School Assessment Package 1

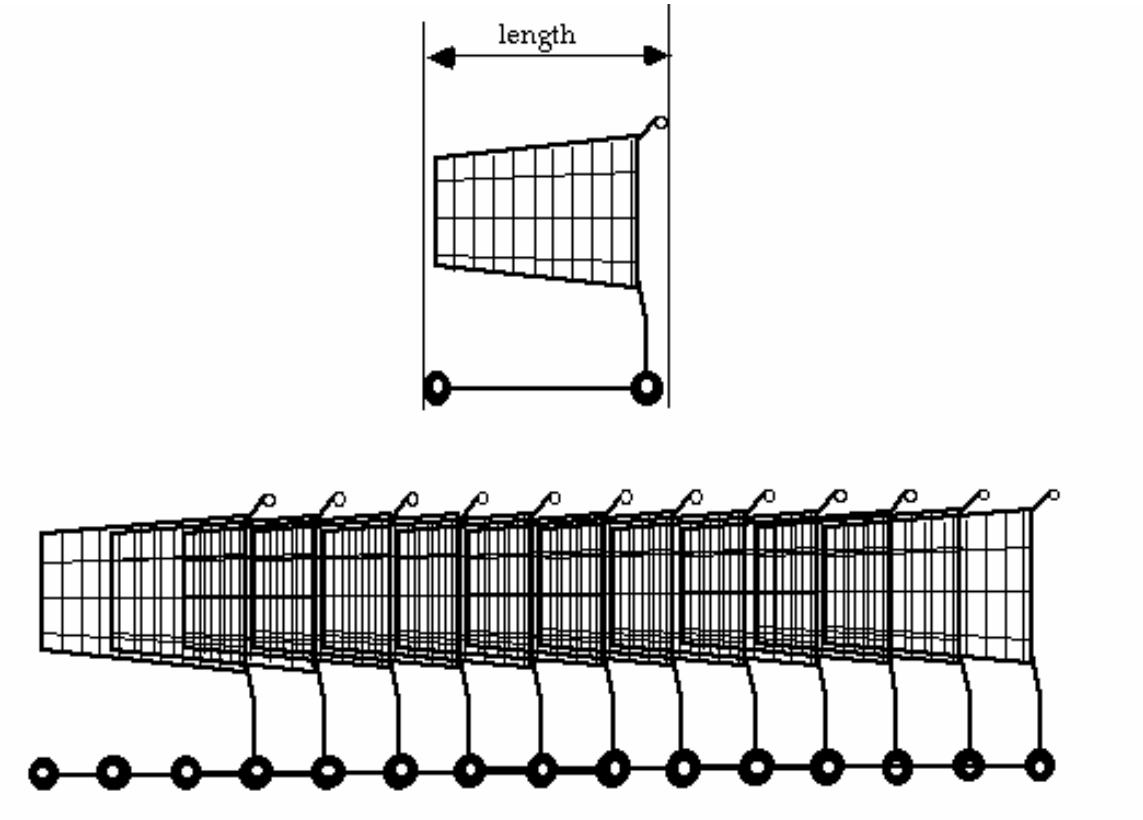
Supermarket Carts

The aim of this assessment is to:

- think mathematically about supermarket carts;
- create a rule that can be used to predict the length of storage space needed, given the number of carts.

The diagram below shows a drawing of a single supermarket cart.
It also shows a drawing of 12 supermarket carts that have been "nested" together.

The drawings are **one twenty-fourth (1/24th) real size**.



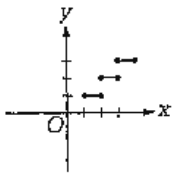
1. Create a rule that will tell you the length of storage space (S) needed when all you know is the number of supermarket carts to be stored.
You will need to show *HOW* you built your rule; that is, we will need to know what data you drew upon and how you used it.

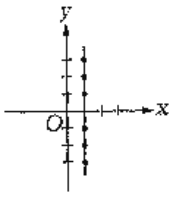
2. Now show how you can figure out the number of carts that can fit in a space S meters long.

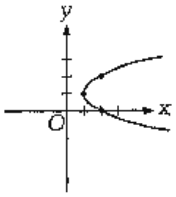
M(F&A)–10–2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).

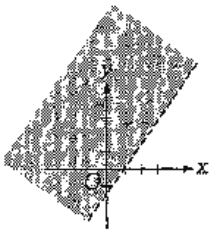
35

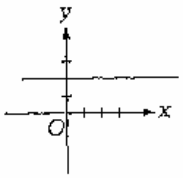
9. Which of the following could be the graph of a function?

A) 

B) 

C) 

D) 

E) 

NAEP Released Item – 1996

M(F&A)–10–2 continued on next page

M(F&A)–10–2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).

Continued…

36

Toilet Graph

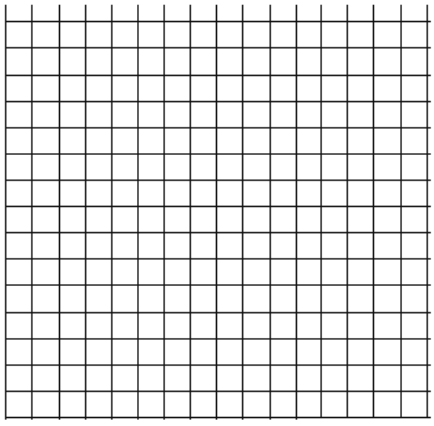
The aim of this assessment is to provide you with the opportunity to:

- Sketch a graph of the way a quantity varies with time.

From the MARS web site – Balanced Assessment for the Mathematics Curriculum - Senior Assessment Package 2

In the situation below, identify a quantity that varies with time. (There may be more than one interesting choice.) Sketch a graph showing **how** the quantity varies with time:

The water tank of a toilet is full. Someone flushes the toilet. The tank refills.



M(F&A)–10–2 continued on next page

M(F&A)–10–2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).

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Continued…

Use the table below to answer question 8.

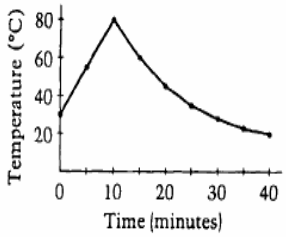
	A	B	C	D	E
1	2.5	5	10	20	*x*
2	20	10	5	2.5	*y*

The table shows part of a computer spreadsheet. The value of each number in row 2 depends on the value of the number above it in row 1.

Write an equation that shows the correct relationship between *x* and *y*.

Adapted WYCAS Released Item – 2003

38



2. The graph above best conveys information about which of the following situations over a 40-minute period of time?

A) Oven temperature while a cake is being baked
B) Temperature of water that is heated on a stove, then removed and allowed to cool
C) Ocean temperature in February along the coast of Maine
D) Body temperature of a person with a cold
E) Temperature on a July day in Chicago

Did you use the calculator on this question?

○ Yes ○ No

NAEP Released Item – 1992

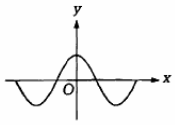
M(F&A)–10–2 continued on next page

M(F&A)–10–2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).

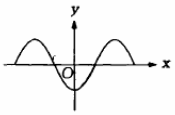
39

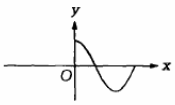
Continued…

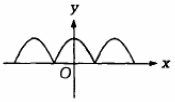
NAEP Released Item – 1992

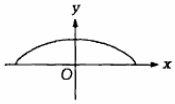


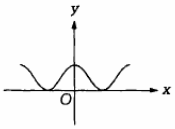
8. The figure above shows the graph of $y = f(x)$. Which of the following could be the graph of $y = |f(x)|$?

A)


B)


C)


D)


E)


40

Use the table below to answer question 14.

NHEIAP Released Item – 2003

Time (hours)	Water Temperature (°F)
0	72
3	87
5	97
10	122

14. This table shows the temperature of water being heated in an industrial water heater. What is the average rate of increase of the temperature in degrees per hour?

A. 5 degrees per hour
B. 12.2 degrees per hour
C. 12.5 degrees per hour
D. 15 degrees per hour

M(F&A)–10–2 continued on next page

M(F&A)–10–2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).

Continued…

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NHEIAP Released Item – 2003

15. Dave and Jeff are planning to drive from New Hampshire to Ohio, a distance of 780 miles. The road atlas gives an estimated driving time that is based on an overall average speed of 50 miles per hour. About how much **less** time will it take them if Dave and Jeff average 60 mph instead of 50 mph for the whole trip?

A. $1\frac{1}{2}$ hours

B. $2\frac{1}{2}$ hours

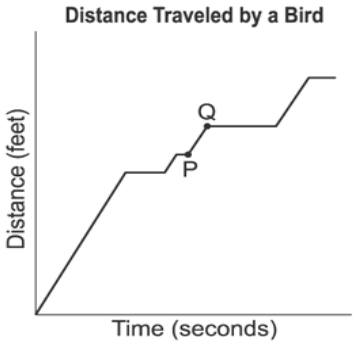
C. 4 hours

D. 8 hours

42

NHEIAP Released Item – 2003

Use the graph below to answer question 28.



28. An ornithologist observed a bird's travel over a period of one minute. She used the data to construct this graph. The **slope** of the graph between points P and Q represents

A. the distance between points P and Q.

B. the number of seconds the bird took to travel from point P to point Q.

C. the bird's rise in height from point P to point Q.

D. the bird's rate of speed between points P and Q.

M(F&A)–10–2 continued on next page

M(F&A)–10–2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).

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Continued…

NHEIAP Released Item – 2001

28. If $x = \frac{4y}{3}$, what happens to y as x increases?

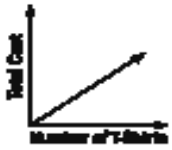
✱ A. y increases
B. y decreases
C. y remains the same
D. You can't tell.

44

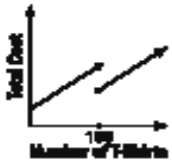
NHEIAP Released Item – 2001

31. This graph shows the relationship between the number of T-shirts printed at a local shop and the total cost. The total cost includes a charge for the initial design and a charge for each T-shirt. If the shop decided to make the initial design free for orders of 100 shirts or more, which graph would illustrate the new price structure?

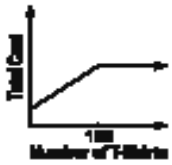
A.



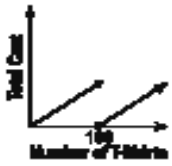
✱ B.



C.



D.



M(F&A)–10–2 continued on next page

M(F&A)–10–2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).

Continued…

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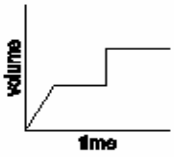
NHEIAP Released Item – 2001

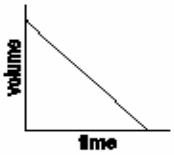
CURRICULUM GOALS 1 & 2

24. A family drinks milk only at breakfast and at dinner. Which graph below best represents the volume of milk remaining in a gallon jug over a two-day period?

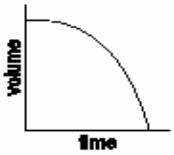
A.



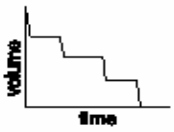
B.



C.



* D.



M(F&A)–10–3 **Demonstrates conceptual understanding of algebraic expressions** by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions.

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7. If x and y are integers, then the expression $4x + 5y$ has a value that is odd or even depending on the values of x and y. For example, if x and y are each even, $4x$ is even and $5y$ is even. Therefore, $4x + 5y$ is also even. Fill in each of the blank spaces in the following table with either "odd" or "even" for the value of $4x + 5y$.

Value of x	Value of y	Value of $4x + 5y$
even	even	even
even	odd	
odd	even	
odd	odd	

NAEP Released Item – 1996

47

4. If $a = 2$ and $b = -1$, what is the value of the expression $\dfrac{a-b}{\frac{1}{a^2}+\frac{1}{b^2}}$?

   A. $\frac{12}{5}$

   B. $\frac{15}{4}$

   C. -4

   D. $\frac{4}{5}$

NHEIAP Released Item – 2004

M(F&A)–10–3 continued on next page

M(F&A)–10–3 **Demonstrates conceptual understanding of algebraic expressions** by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions.

48

Continued…

LICHEN

Adapted PISA Sample Item – 2003

A result of global warming is that the ice of some glaciers is melting. Twelve years after the ice disappears, tiny plant-like fungus, called lichen, start to grow on the rocks.

Each lichen grows approximately in the shape of a circle.

The relationship between the diameter of this circle and the age of the lichen can be approximated with the formula:

$$d = 7.0 \times \sqrt{(t - 12)} \quad \text{for } t \geq 12$$

where d represents the diameter of the lichen in millimeters, and t represents the number of years after the ice has disappeared.

Using the formula, calculate the diameter of the lichen, 16 years after the ice disappeared. Show your calculation.

49

27. One leg of a right triangle is twice as long as the other leg. If the length of the shorter leg is x, which expression represents the area of the triangle?

NHEIAP Released Item – 2003

A. x^2

B. $2x^2$

C. $2x$

D. $\frac{1}{2}x$

M(F&A)–10–3 continued on next page

M(F&A)–10–3 **Demonstrates conceptual understanding of algebraic expressions** by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions.

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Continued…

NHEIAP Released Item – 2003

6. The temperature at the top of Mt. Washington one morning was $-10°$ Fahrenheit. With the conversion formula $C = \frac{5}{9}(F - 32)$, what was the temperature in degrees Celsius (to the nearest degree)?

   A. $-48°$
   B. $-28°$
   C. $-23°$
   D. $-12°$

M(F&A)–10–4 **Demonstrates conceptual understanding of equality** by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations.

51

LICHEN

Adapted PISA Sample Item – 2003

A result of global warming is that the ice of some glaciers is melting. Twelve years after the ice disappears, tiny plant-like fungus, called lichen, start to grow on the rocks.

Each lichen grows approximately in the shape of a circle.

The relationship between the diameter of this circle and the age of the lichen can be approximated with the formula:

$$d = 7.0 \times \sqrt{(t - 12)} \quad \text{for } t \geq 12$$

where *d* represents the diameter of the lichen in millimeters, and *t* represents the number of years after the ice has disappeared.

Ann measured the diameter of some lichen and found it was 35 millimeters.

How many years ago did the ice disappear at this spot?

Show your calculation.

M(F&A)–10–4 continued on next page

M(F&A)–10–4 **Demonstrates conceptual understanding of equality** by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations.

Continued…

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$4 \times \square = \square$ and $\square \times 3 = \square$

6. What number if placed in each box above would make both equations true?

NAEP Released Item – 1996

A) 0
B) 1
C) 2
D) 3
E) 4

53

32. Mr. Johnson purchased 20 concert tickets for a total of $225. The concert tickets cost $15 for adults and $10 for children under 12. How many tickets for children under 12 did Mr. Johnson purchase?

Adapted MCAS Released Item – 2004

54

42 A local ski club plans to charter transportation for a ski trip. Two different bus companies are available for charter services.

SNOWBIRD CHARTER	MOUNTAIN CHARTER
Roundtrips Depart Daily 6 and 8 A.M.	*Roundtrips Daily at 6 and 8 A.M.*
$300, plus $12 per person	*$15 per person*
Reservations are required.	**Call for reservations.**

If the club members want to choose the less expensive of the two bus companies, which company should they choose? Justify your answer by explaining how the number of club members who go on the trip should affect their decision.

Adapted MCAS Released Item – 2004

M(F&A)–10–4 continued on next page

M(F&A)–10–4 **Demonstrates conceptual understanding of equality** by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations.

Continued…

55

26. A florist plans to spend about $400 on daffodils and tulips. Tulips cost $8 per dozen, and daffodils cost $5 per dozen. If the florist expects to sell twice as many daffodils as tulips, how many dozens of daffodils should he order?

    A. 66
    B. 44
    C. 33
    D. 22

NHEIAP Released Item – 2003

56

Use the information in the box below to answer question 10.

From January through October, Midway Autos sold 150 cars. The number of cars sold in November and the number of cars sold in December were in the ratio 3 : 5. If the total number of cars sold in the year was 390, find the number of cars sold in December.

10. Which equation can be used to solve the problem above?

    A. $3x + 5x + 150 = 390$
    B. $150 - 8x = 390$
    C. $8x = 390$
    D. $3x - 5y = 240$

NHEIAP Release Item – 2003

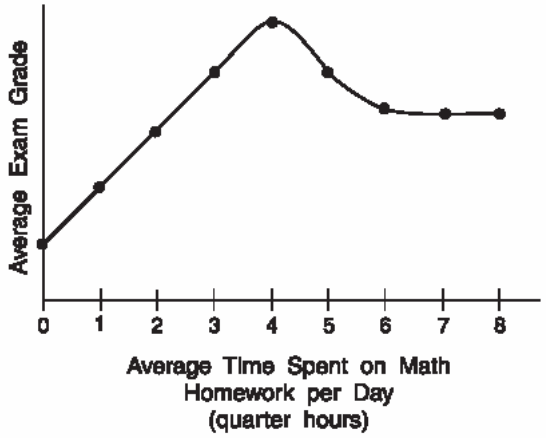
M(DSP)–10–1 **Interprets a given representation(s)** (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g., media, workplace, social and environmental situations).

57

(IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–10–2*.)

NHEIAP Released Item – 2001

38. A researcher asked many students these two questions: "What was your grade on your last math exam?" and "How many hours per night (to the nearest quarter hour) do you usually spend on math homework?" The researcher then sorted the data into groups according to how much time was spent on homework. Finally, the researcher computed an average exam grade for each of these groups and plotted the averages to produce the graph below.



Interpret the graph by describing how average exam grades are related to time spent studying. Be complete.

M(DSP)–10–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts** by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of **the sample** from which the statistics were developed.

NAEP Released Item – 1996

58

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.

10. The table below shows the daily attendance at two movie theaters for 5 days and the mean(average) and the median attendance.

	Theater A	Theater B
Day 1	100	72
Day 2	87	97
Day 3	90	70
Day 4	10	71
Day 5	91	100
Mean (average)	75.6	82
Median	90	72

(a) Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater A? Justify your answer.

(b) Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater B? Justify your answer.

59

14. Ms. Gill gave a mathematics test to her class. The grade statistics were as follows:
    median = 83
    mean = 84
    low score = 67
    high score = 98

WYCAS Released Item – 2003

After Ms. Gill handed back the graded tests, she realized that one student who was given a score of 94 should have received a 99. Which of the following statistics will **not** increase as a result of this change?

A. the median
B. the mean
C. the range
D. All three will increase.

M(DSP)–10–2 continued on next page

M(DSP)–10–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts** by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of **the sample** from which the statistics were developed.

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Continued…

Adapted from a Balanced Assessment Task, High School Assessment Package 2 – VSMIT

Ordering a Cab

Sarah has to take a cab to school each day. Two companies are available: Sunshine Cabs and Bluebird Cabs. Sarah conducts a survey over several months to compare the two cab companies. She orders each taxi 20 times and records how early or late they are when arriving to pick her up.

Sunshine Cabs		Bluebird Cabs	
3 min. 30 sec.	Early	3 min. 45 sec.	Late
45 sec.	Late	4 min. 30 sec.	Late
1 min. 30 sec.	Late	3 min.	Late
4 min. 30 sec.	Late	5 min.	Late
45 sec.	Early	2 min. 15 sec.	Late
2 min. 30 sec.	Early	2 min. 30 sec.	Late
4 min. 45 sec.	Late	1 min. 15 sec.	Late
2 min. 45 sec.	Late	45 sec.	Late
30 sec.	Late	3 min.	Late
1 min. 30 sec.	Early	30 sec.	Early
2 min. 15 sec.	Late	1 min. 30 sec.	Late
9 min. 15 sec.	Late	3 min. 30 sec.	Late
3 min. 30 sec.	Late	6 min.	Late
1 min. 15 sec.	Late	4 min. 30 sec.	Late
30 sec.	Early	5 min. 30 sec.	Late
2 min. 30 sec.	Late	2 min. 30 sec.	Late
30 sec.	Late	4 min. 15 sec.	Late
7 min. 15 sec.	Late	2 min. 45 sec.	Late
5 min. 30 sec.	Late	3 min. 45 sec.	Late
3 min.	Late	4 min. 45 sec.	Late

1. Analyze the data, and then use the data to present a reasoned case for Sunshine Cabs being the better company for Sarah to use in the future. Present your reasoning as fully and as clearly as possible.

2. Analyze the same data to present a reasoned case for Bluebird Cabs being the better company. Again, present your reasoning as fully and clearly as possible.

3. Which argument do you think is more convincing? Substantiate your reasoning with data.

M(DSP)–10–2 continued on next page

M(DSP)–10–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts** by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of **the sample** from which the statistics were developed.

Continued…

61

Construct a data set of 10 exam scores so that the difference between the upper and lower quartile equals zero and the mean is less than the median.

Adapted ARTIST Item ID = Q0175
MEASURES OF CENTER; MEASURES OF SPREAD
COMPUTATION REASONING
Released Item – Last Modified **March 1, 2004**

62

17. Which of the following is most sensitive to outliers?

a. interquartile range
b. mean
c. median
d. mode

ARTIST Item ID = Item ID = Q1076
MEASURES OF SPREAD
REASONING Released Item – Last Modified
March 1, 2004
NSF-funded Web ARTIST project

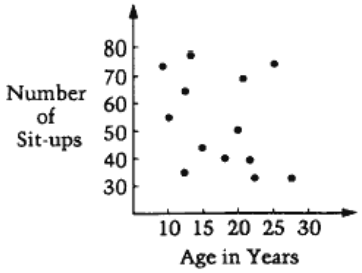
M(DSP)–10–2 continued on next page

M(DSP)–10–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts** by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of **the sample** from which the statistics were developed.

Continued…

63

NAEP Released Item – 1992



3. In the graph above, each dot shows the number of sit-ups and the corresponding age for one of 13 people. According to this graph, what is the median number of sit-ups for these 13 people?

A) 15
B) 20
C) 45
D) 50
E) 55

64

To assess the opinion of employees at a software company a researcher interviews the first fifteen employees who are willing to express their opinion. Explain why this does or does not represent a random sample?

Adapted ARTIST Item ID = Q1070
DATA PRODUCTION LITERACY Released Item – Last Modified March 1, 2004

M(DSP)–10–2 continued on next page

M(DSP)–10–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts** by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of **the sample** from which the statistics were developed.

65

Continued…

31 In an experiment, Sue and Helise asked each of 30 students in a random sample of the juniors at their school to record the number of minutes they watched television on a Saturday and Sunday in April. The results, rounded to the nearest 30 minutes, are shown in the table.

Minutes of Television Watching

Total Number of Minutes of Television Watched on Saturday and Sunday	Number of Junior Students
0	1
60	3
90	6
120	5
180	5
240	2
300	5
420	1
540	2

a. What number of minutes spent watching television should the girls report as the mode for this group of students? Justify your answer.

b. Helise said that the median number of minutes for this group of students is 180, but Sue disagreed. Do you agree with Sue or Helise? Justify your answer.

c. Suppose that Sue and Helise had used the entire class of 185 juniors as their sample. Based on the results from their smaller sample, what total number of the 185 juniors would probably have reported watching 300 minutes of television on that weekend? Show or explain how you obtained your answer.

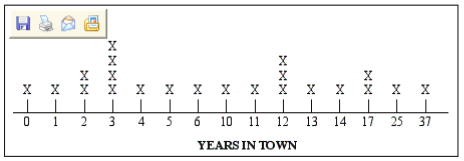
MCAS Released Item – 2004

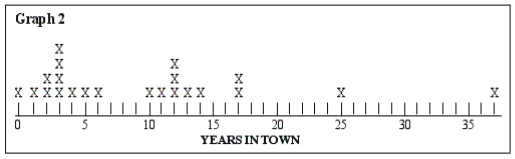
M(DSP)–10–3 **Identifies or describes representations or elements of representations that best display a given set of data or situation**, consistent with the representations required in M(DSP)–10–1.

66

ARTIST Released Item Item ID = Q0108 Data Representation Reasoning – Last Modified March 1, 2004

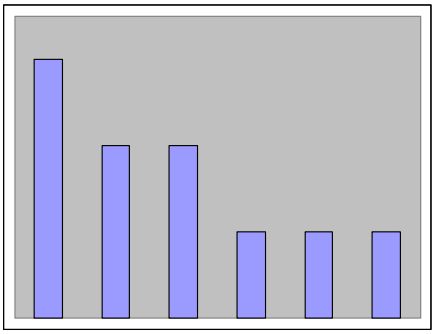
1. A class of students recorded the number of years their families had lived in their town. Here are two graphs that students drew to summarize the data. Which graph gives a more accurate representation of the data? Why?

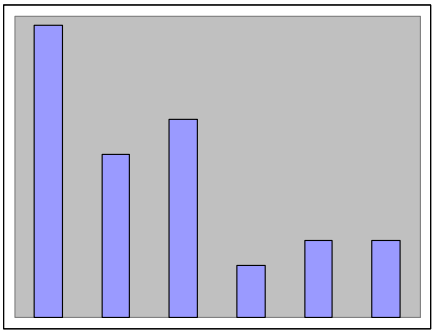




67

M&M Candies reports that the plain M&Ms are manufactured with 30% brown, 20% red, 20% yellow, 10% orange, 10% blue, and 10% green candies. Below are 2 graphs. Identify which graph represents the colors of individual M&Ms for the population of all M&M Candies, and which graph represents the colors in a sample of 20 candies. Provide a rationale for your choices.





Adapted from ARTIST Item ID = Q0845
DATA REPRESENTATION REASONING
Released Item – Last Modified **March 1, 2004**

M(DSP)–10–4 **Uses counting techniques to solve problems** in context involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or[sc] others).

Adapted From the MARS web site – Balanced Assessment for the Mathematics Curriculum - Senior Assessment Package 2

Dog Tags

1. You would like to name your new pet terrier, but wish to come up with something more creative than **Rover**. At the same time, you don't want to choose a name so exotic that it can't be pronounced, like **Xwvrq**. You notice that most names where vowels and consonants alternate (like **Azure** or **Bozon**) can be pronounced. The question is, how many different names could you make up, subject to the following rules:

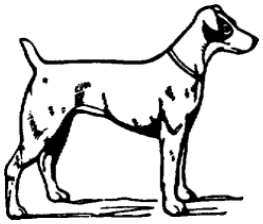
- The name must be 5 letters long.
- It can start with either a vowel or a consonant.
- Vowels and consonants must alternate. (Consider **y** to always be a consonant, so there are 5 vowels and 21 consonants.)
- You may use the same letter more than once.
- **Rover** is definitely out of the question!

2. Your whimsical little brother thinks it would be cute to choose a name beginning with **T**, for alliterative purposes (e.g. "**Tarok** the Terrier"). What portion of your choices would this constraint eliminate? (State your answer as both a fraction and a percentage.)

3. The humane society gave you a kit for making an identification tag bearing your dog's name. Unfortunately, it contains only one of each letter of the alphabet, which wouldn't allow for names like **Rover** (which you had ruled out anyway) or **Bozon**. Determine how serious a restriction it is to disallow repeated letters, by answering the following questions:

a. Out of your original list of names (from question **1**), what portion is eliminated?

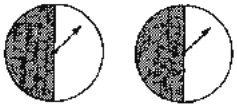
b. Out of the names your brother would like (just the ones beginning with a **T**, from question **2**), what portion is eliminated?



M(DSP)–10–5 **Solves problems involving experimental or theoretical probability.**

69

NAEP Released Item – 1996



9. The two fair spinners shown above are part of a carnival game. A player wins a prize only when both arrows land on black after each spinner has been spun once.

James thinks he has a 50-50 chance of winning. Do you agree?

Ⓐ Yes Ⓑ No

Justify your answer.

70

15. Philip is doing a probability experiment with two cubes with sides numbered 1 through 6. He tosses the cubes and adds the numbers that land facing up. One cube lands with a 1 facing up. What is the probability that the sum of the two numbers will be 6 or more?

Adapted WYCAS Released Item – 2003

A. $\frac{4}{11}$

B. $\frac{1}{3}$

C. $\frac{2}{11}$

D. $\frac{1}{6}$

71

Dave rolls two number cubes with sides numbered 1through 6. After each roll he records the difference of the two numbers that land face up. If the numbers are different, he always subtracts the smaller number from the larger number. Which difference is Dave most likely to get?

A. 4
B. 3
C. 2
D. 1

Item Created by RI Teachers at Prime Time Institute, 2005 (wording slightly adapted)

Appendix A
Item Links

This Appendix contains web links to the primary pages where the samples items that populate this document are located (with the exception of a couple of items taken from publications as noted in their item reference). Many of these sites contain additional materials such as scoring rubrics, exemplars, answers, and performance data.

Assessment Resource Tools for Improving Statistical Thinking (ARTIST): https://ore.gen.umn.edu/artist/

National Assessment of Education Progress (NAEP) Question Tool:
http://nces.ed.gov/nationsreportcard/itmrls/

Massachusetts Comprehensive Assessment System (MCAS): http://www.doe.mass.edu/mcas/

Mathematics Assessment Resource Service (MARS):
http://www.nottingham.ac.uk/education/MARS/tasks/

New Hampshire Educational Improvement and Assessment Program (NHEIAP):
http://www.ed.state.nh.us/education/doe/organization/curriculum/Assessment/NHIAP.htm

Programme for International Student Assessment (PISA):
http://www.pisa.oecd.org/pages/0,2966,en_32252351_32236130_1_1_1_1_1,00.html

The Vermont Institute for Science, Math, and Technology (VISMT):
http://www.vermontinstitutes.org/pub/ProblemSolve.pdf

Wyoming Comprehensive Assessment System (WYCAS):
http://www.k12.wy.us/eqa/aa/programs/wycas.asp

Appendix B
Depth of Knowledge Descriptors for Mathematics
Norman L. Webb
March 28, 2002

Mathematics depth-of-knowledge levels

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

Appendix C
Preliminary Depth of Knowledge Levels for Mathematics GSEs

	Depth of Knowledge Levels
Number and Operation	
M(N&O)–10–1	
M(N&O)–10–2	1, 2, 3
M(N&O)–10–3	
M(N&O)–10–4	1, 2, 3
M(N&O)–10–5	
M(N&O)–10–6	
M(N&O)–10–7	
M(N&O)–10–8	
Geometry and Measurement	
M(G&M)–10–1	
M(G&M)–10–2	1, 2, 3
M(G&M)–10–3	
M(G&M)–10–4	2, 3
M(G&M)–10–5	1, 2, 3
M(G&M)–10–6	1, 2, 3
M(G&M)–10–7	1, 2
M(G&M)–10–8	
M(G&M)–10–9	2,3
M(G&M)–10–10	
Functions and Algebra	
M(F&A)–10–1	2, 3
M(F&A)–10–2	1, 2, 3
M(F&A)–10–3	1, 2
M(F&A)–10–4	1, 2, 3
Data, Statistics, and Probability	
M(DSP)–10–1	2, 3
M(DSP)–10–2	
M(DSP)–10–3	1, 2, 3
M(DSP)–10–4	1, 2, 3
M(DSP)–10–5	
M(DSP)–10–6	

Black – No Assessment GSE at this level

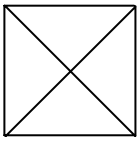
Appendix D
Preliminary Distribution of Emphasis

Distribution of Emphasis by content strands grades 3–8, and Preliminary RI/NH 9–10 Mathematics GSEs (percent of score points on assessment from each content strand)

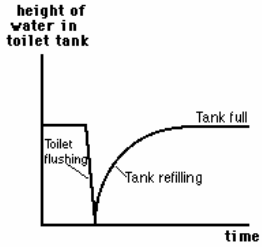
Note: 2(3) means grade 2 GLEs assessed in the beginning of grade 3

	Approximate Percent Distribution of Emphasis						
	2 (3)	**3(4)**	**4(5)**	**5(6)**	**6(7)**	**7(8)**	**9–10 GSEs**
Number and Operations	55%	50%	50%	45%	30%	20%	15%
Geometry and Measurement	15%	20%	20%	25%	25%	25%	30%
Functions and Algebra	15%	15%	15%	15%	30%	40%	40%
Data, Statistics, and Probability	15%	15%	15%	15%	15%	15%	15%
	100%	100%	100%	100%	100%	100%	100%

Appendix E
Answers

Item Number	Answer	Depth of Knowledge Level
1	B	2
2	D	2
3	e; d; g; f; g; reasons may vary.	2
4	C	1
5	Martin's drink; Sample answer: Luis adds 1 ounce of Cherry Soda to every 53/6 or about 8.8 ounces of water , whereas Martin adds 1 ounce of Cherry Soda to every 42/5 = 8.4 ounces of water	2
6	B	2
7	D	2
8	on its way up	2
9	B	2
10	Yes; Sample answer: If k represent a person's income, than we are looking for k such that $0.05k = (k - 10{,}000)*0.06$. Solving for k we get k = \$60,000; No; Sample justification: If a person's effective tax rate were 6% than that would imply that 6% of the person's income is the same as 6% of \$10,000 less than the person's income. This can never happen. As a person's income increases subtracting \$10,000 from the income does not have much affect on the income, so 6% of the person's income will be close to 6% of \$10,000 less than the person's income, however these numbers will never be equal.	3
11	C	2
12	Sample answer: Fold the paper into semi-circles two different ways to create two diameters. Since the diameters must pass through the center of the circle, their intersection point is the center.	2
13	Sample answer: 	1
14	D	1
15	D	1
16	B	2
17	B	2
18	B	2
19	Sample Answer: The two triangles are similar since angles A and P both measure 60 degrees, angles B and Q both measure 80 degrees, and angles C and R both measure 40 degrees; $3 + x + y$, $12 + 4x + 4y$; 1/16; Sample answer: the ratio of corresponding sides is 1:4 and the area involves the produce of the base and heights, so the ratio of the areas is 1:16.	2
20	D	2
21	A	2

22	Shape B; Sample answer: Shape B has the largest area since the other shapes will fit inside of it; Sample answer: Place Shape C on a grid and count the number of squares on the grid contained in the shape and the number of square containing some part of the shape. This will give an underestimate and overestimate for the area. To achieve a better estimate, subdivide each of the original squares on the grid into four equal size pieces. Then repeat the process. Continuing this process will improve the estimate for the area of Shape C; Sample answer: Trace Shape C with a string and then measure the length of the string used.	2
23	≈ 282.74 cm^3; Sample answer: $V = \pi r^2 h = \pi(3)^2(10) \approx 282.74$ cm^3; ≈ 28.27 cm^3; The increase in volume is 1/10 of the volume found in part a; ≈ 5.65 cm^3; Sample answer: The volume of 1 marble is 1/5 of the increase in volume found in part b; ≈ 1.10 cm; Sample answer: The volume of a marble is given by $V = \frac{4}{3}\pi r^3$. Since V is about 5.65 (from part c), multiplying by 3 and dividing by 4π and then taking the cube root gives about 1.10 cm.	2
24	less than; Sample answer: The volume of each cylinder is $\pi r^2 h$, where h represents the height of the cylinder. The radius of Cylinder A is $\frac{w}{2\pi}$ since the circumference of the cylinder is w. Similarly, the radius of Cylinder B is $\frac{l}{2\pi}$. Since $l > w$, we have $l^2 w > w^2 l$. Therefore, the volume of Cylinder A is less than the volume of Cylinder B.	3
25	B	2
26	30 cm^2	1
27	D	1
28	The volume, V, of the gutter is equal to the area of a semi-circular cross-section times the length of the gutter, L. The area of the semi-circular cross-section is $\frac{1}{2}\pi r^2$, where r is the radius. Since w is the circumference of the semi-circle $w = \pi r$ or $r = \frac{w}{\pi}$. So, $V = \frac{1}{2}\pi(\frac{w}{\pi})^2 L = \frac{w^2 L}{2\pi}$.	3
29	Sample answer: Jay is not correct. Let l, w, and h represent the length, width, and height of Jay's rectangular bag, respectively. Then the volume, V, of his bag is given by $V = lwh$. If Jay uses a bag whose dimensions are half of the original dimensions, then the volume of this new bag is given by $\frac{l}{2} \cdot \frac{w}{2} \cdot \frac{h}{2} = \frac{1}{8} lwh = \frac{1}{8} V$. So, his new bag has one-eighth of the volume of his original bag.	2
30	A	1
31	C	2
32	442; Sample answer: The 20th figure consists of a 20 by 22 rectangle of tiles with two additional tiles – one added to the top row and one added to the bottom row. The nth figure in the pattern consists of a n by n + 2 rectangle with two additional tiles – one added to the top row and one added to the bottom row.	2

<table>
<tr><td>33</td><td>B</td><td>2</td></tr>
<tr><td>34</td><td>[Note: Answer will be dependent upon whether students measure from the computer screen, along with the zoom factor, or if they measure from printed copy.]
$S = 0.96 + 0.28(n - 1)$; Sample answer: 12 nested carts is equal to the length of 1 cart plus 11 times the amount that each cart sticks out from the preceding cart, x. 12 nested carts is about 17 cm on the diagram or 17 * 24 = 408 cm or 4.08 m in real length. The length of 1 cart is 4 cm on the diagram or 4 * 24 = 96 cm or 0.96 m in real length. Therefore, $4.08 = 0.96 + 11(x)$. Solving for x we get about 0.28 m. In general the amount of space needed, S, is equal to the length of one cart plus 0.28 times the number of carts minus 1 or $S = 0.96 + 0.28(n - 1)$; Solving for n in the previous equation gives the number of carts as a function of the storage space,
$n = (\frac{S - 0.96}{0.28}) + 1$</td><td>3</td></tr>
<tr><td>35</td><td>E</td><td>1</td></tr>
<tr><td>36</td><td>Height of water in the tank varies with time; Sample answer (from http://www.nottingham.ac.uk/education/MARS/tasks/g12_1/full.htm):
 

(Note: The essential features of this graph are that it takes the tank longer to refill than to empty. Also, in this graph, if one thought of the tank as refilling at a constant rate it is reasonable that the graph of the height as a function of time while refilling is concave down – showing that the height is increasing at a decreasing rate due to the bowl widening.)</td><td>3</td></tr>
<tr><td>37</td><td>$y = \frac{50}{x}$</td><td>2</td></tr>
<tr><td>38</td><td>B</td><td>2</td></tr>
<tr><td>39</td><td>C</td><td>2</td></tr>
<tr><td>40</td><td>A</td><td>1</td></tr>
<tr><td>41</td><td>B</td><td>2</td></tr>
<tr><td>42</td><td>D</td><td>2</td></tr>
<tr><td>43</td><td>A</td><td>2</td></tr>
<tr><td>44</td><td>B</td><td>2</td></tr>
<tr><td>45</td><td>D</td><td>2</td></tr>
<tr><td>46</td><td><table>
<tr><td>Value of x</td><td>Value of y</td><td>Value of $4x + 5y$</td></tr>
<tr><td>Even</td><td>Even</td><td>Even</td></tr>
<tr><td>Even</td><td>Odd</td><td>Odd</td></tr>
<tr><td>Odd</td><td>Even</td><td>Even</td></tr>
<tr><td>Odd</td><td>Odd</td><td>Odd</td></tr>
</table></td><td>2</td></tr>
<tr><td>47</td><td>A</td><td>1</td></tr>
<tr><td>48</td><td>14 mm; $d = 7\sqrt{16 - 12} = 7\sqrt{4} = 7 * 2 = 14$</td><td>1</td></tr>
<tr><td>49</td><td>A</td><td>2</td></tr>
<tr><td>50</td><td>C</td><td>1</td></tr>
</table>

51	37; $35 = 7\sqrt{t-12}$ or $5 = \sqrt{t-12}$ so $t-12 = 25$ or $t = 37$.	1
52	A	1
53	15	2
54	Let *n* represent the number of people attending the trip. Then the two charters are equal in price when 300 + 12*n* = 15*n* or when *n* = 100. Since 15*n* is less than 300 + 12*n* for *n* less than 100, the club should choose Mountain Charter if less than 100 people plan on attending the trip and they should choose Snowbird Charter if greater than 100 people plan on attending the trip.	2
55	B	2
56	A	2
57	Sample answer: Average grades increased with study time up to 1 hour and then slightly decreased but seemed to level off after 1 ½ hours of studying.	2
58	median; Sample answer: The attendance on day 4 is so low that it makes the mean not as representative of the data as the median; mean; Sample answer: There are two clusters and the median is only representative of one of them, while the mean is a better indicator of where the 'center' of the data is.	3
59	A	2
60	(from http://www.vermontinstitutes.org/pub/ProblemSolve.pdf): Sample answer: In the 20 times she called each company, Sarah had to spend 65 minutes waiting for Bluebird Cabs, while she only spent 49.75 minutes waiting for Sunshine Cabs. So she might want to go with Sunshine Cabs; Sample answer: On the other hand, whereas Sunshine Cabs are, on average, not as late, they are less consistent in their arrival times than Bluebird Cabs. It may therefore be better to book with Bluebird Cabs, making sure that the cab is ordered for about 5 minutes before it is needed; answers will vary. (See the web site for a sample approach.)	3
61	Sample answer: 10, 10, 80, 80, 80, 80, 80, 80, 80, 80	2
62	B	2
63	D	2
64	does not represent a random sample; sample answer: Since the researcher only interviewed the first 15 people who were willing to express their opinions, it may be the case that these opinions are very strong (very positive, negative, or both) and may not be representative of the opinions of all employees.	2
65	90 minutes; Sample answer: the mode is the number of minutes occurring most frequently; Sue is correct; Sample answer: 10 people watched 90 minutes or less and 10 people watched 240 minutes or more with 5 people watching 120 minutes and 5 people watching 180 minutes. Therefore, the median is the average of 120 and 180, or 150 minutes; about 31 juniors; Sample answer: 1/6 of the 30 juniors in the sample reported watching 300 minutes of television, so we would expect about 1/6 of 185 juniors or about 31 juniors would report watching 300 minutes of television on that weekend.	2
66	Graph 2; Sample answer: The scale on Graph 1 isn't consistent; it only includes years that have data points associated with them. This causes the graph to be misleading; it does not accurately show the spread of the data.	2

67	The graph on the left represents the population and the one on the right represents the sample. This is evident by the two bars on the left graph of equal height representing 20% and the three bars on the left graph of equal height representing 10% (the actual percentages for the population); whereas the graph on the right is indicative of a sample – the percentages vary from the population percentages slightly.	2
68	286,649; about 96%; about 36%; about 31%	3
69	James is incorrect; Sample answer: Since each spinner has one black and one white region each spinner could land on one of two possible colors resulting in four possibilities (BB, BW, WB, WW) of which only one is black on both spinners. So, James has a ¼ chance of winning.	1
70	A	2
71	D	2